8-5 Practice

Angles of Elevation and Depression

Name the angle of depression or angle of elevation in each figure.

1. \( \angle TRZ; \angle YZR \)

2. \( \angle PRM; \angle LMR \)

3. **WATER TOWERS** A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth degree.

   about 13.5°

4. **CONSTRUCTION** A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55°, how far is the ladder from the base of the wall? Round your answer to the nearest foot.

   about 21 ft

5. **TOWN ORDINANCES** The town of Belmont restricts the height of flagpoles to 25 feet on any property. Lindsay wants to determine whether her school is in compliance with the regulation. Her eye level is 5.5 feet from the ground and she stands 36 feet from the flagpole. If the angle of elevation is about 25°, what is the height of the flagpole to the nearest tenth foot?

   about 22.3 ft

6. **GEOGRAPHY** Stephan is standing on a mesa at the Painted Desert. The elevation of the mesa is about 1380 meters and Stephan’s eye level is 1.8 meters above ground. If Stephan can see a band of multicolored shale at the bottom and the angle of depression is 29°, about how far is the band of shale from his eyes? Round to the nearest meter.

   about 2850 m

7. **INDIRECT MEASUREMENT** Mr. Dominguez is standing on a 40-foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?

   about 27 ft
1. **LIGHTHOUSES** Sailors on a ship at sea spot the light from a lighthouse. The angle of elevation to the light is 25°.

   The light of the lighthouse is 30 meters above sea level. How far from the shore is the ship? Round your answer to the nearest meter.

   64 m

2. **RESCUE** A hiker dropped his backpack over one side of a canyon onto a ledge below. Because of the shape of the cliff, he could not see exactly where it landed.

   From the other side, the park ranger reports that the angle of depression to the backpack is 32°. If the width of the canyon is 115 feet, how far down did the backpack fall? Round your answer to the nearest whole number.

   72 ft

3. **AIRPLANES** The angle of elevation to an airplane viewed from the control tower at an airport is 7°. The tower is 200 feet high and the pilot reports that the altitude is 5200 feet. How far away from the control tower is the airplane? Round your answer to the nearest foot.

   41,028 ft

4. **PEAK TRAM** The Peak Tram in Hong Kong connects two terminals, one at the base of a mountain, and the other at the summit. The angle of elevation of the upper terminal from the lower terminal is about 15.5°. The distance between the two terminals is about 1365 meters. About how much higher above sea level is the upper terminal compared to the lower terminal? Round your answer to the nearest meter.

   365 m

5. **HELICOPTERS** For Exercises 5–7, use the following information.
   Jermaine and John are watching a helicopter hover above the ground.

   Jermaine and John are standing 10 meters apart.

   5. Find two different expressions that can be used to find the $h$, height of the helicopter.

   \[ h = x \tan 55°; \quad h = (x + 10)\tan 48° \]

   6. Equate the two expressions you found for Exercise 5 to solve for $x$. Round your answer to the nearest hundredth.

   34.98 m

   7. How high above the ground is the helicopter? Round your answer to the nearest hundredth.

   49.95 m
Study Guide and Intervention
The Law of Sines

The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

| Law of Sines | \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \) |

Example 1 In \( \triangle ABC \), find \( b \).

\[
\begin{align*}
\sin C &= \frac{\sin B}{b} \\
\sin 45^\circ &= \frac{\sin 74^\circ}{b} \\
b \sin 45^\circ &= 30 \sin 74^\circ \\
b &= \frac{30 \sin 74^\circ}{\sin 45^\circ} \\
b &\approx 40.8 \\
\end{align*}
\]

Example 2 In \( \triangle DEF \), find \( m \angle D \).

\[
\begin{align*}
\frac{\sin D}{d} &= \frac{\sin E}{e} \\
\frac{28}{d} &= \frac{58^\circ}{24} \\
24 \sin D &= 28 \sin 58^\circ \\
\sin D &= \frac{28 \sin 58^\circ}{24} \\
D &= \sin^{-1} \left( \frac{28 \sin 58^\circ}{24} \right) \\
D &\approx 81.6^\circ
\end{align*}
\]

Exercises

Find each measure using the given measures of \( \triangle ABC \). Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If \( c = 12 \), \( \angle A = 80^\circ \), and \( \angle C = 40^\circ \), find \( a \).
   \[ 18.4 \]

2. If \( b = 20 \), \( c = 26 \), and \( \angle C = 52^\circ \), find \( \angle B \).
   \[ 37 \]

3. If \( a = 18 \), \( c = 16 \), and \( \angle A = 84^\circ \), find \( \angle C \).
   \[ 62 \]

4. If \( a = 25 \), \( \angle A = 72^\circ \), and \( \angle B = 17^\circ \), find \( b \).
   \[ 7.7 \]

5. If \( b = 12 \), \( \angle A = 89^\circ \), and \( \angle B = 80^\circ \), find \( a \).
   \[ 12.2 \]

6. If \( a = 30 \), \( c = 20 \), and \( \angle A = 60^\circ \), find \( \angle C \).
   \[ 35 \]
8-6 Study Guide and Intervention (continued)

The Law of Sines

Use the Law of Sines to Solve Problems You can use the Law of Sines to solve some problems that involve triangles.

| Law of Sines | Let \( \triangle ABC \) be any triangle with \( a, b, \) and \( c \) representing the measures of the sides opposite the angles with measures \( \angle A, \angle B, \) and \( \angle C, \) respectively. Then \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}. \) |

**Example**

Isosceles \( \triangle ABC \) has a base of 24 centimeters and a vertex angle of 68°. Find the perimeter of the triangle.

The vertex angle is 68°, so the sum of the measures of the base angles is 112 and \( m\angle A = m\angle C = 56. \)

\[
\begin{align*}
\frac{\sin B}{b} & = \frac{\sin A}{a} \\
\frac{\sin 68^\circ}{24} & = \frac{\sin 56^\circ}{a} \\
\sin 68^\circ & = 24 \sin 56^\circ \\
a & = \frac{24 \sin 56^\circ}{\sin 68^\circ} \\
& \approx 21.5
\end{align*}
\]

Cross multiply. Use a calculator.

The triangle is isosceles, so \( c = 21.5. \)

The perimeter is \( 24 + 21.5 + 21.5 \) or about 67 centimeters.

**Exercises**

Draw a triangle to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. One side of a triangular garden is 42.0 feet. The angles on each end of this side measure 66° and 82°. Find the length of fence needed to enclose the garden.
   \[ 192.9 \text{ ft} \]

2. Two radar stations \( A \) and \( B \) are 32 miles apart. They locate an airplane \( X \) at the same time. The three points form \( \angle XAB, \) which measures 46°, and \( \angle XBA, \) which measures 52°. How far is the airplane from each station?
   \[ 25.5 \text{ mi from } A; 23.2 \text{ mi from } B \]

3. A civil engineer wants to determine the distances from points \( A \) and \( B \) to an inaccessible point \( C \) in a river. \( \angle BAC \) measures 67° and \( \angle ABC \) measures 52°. If points \( A \) and \( B \) are 82.0 feet apart, find the distance from \( C \) to each point.
   \[ 86.3 \text{ ft to point } B; 73.9 \text{ ft to point } A \]

4. A ranger tower at point \( A \) is 42 kilometers north of a ranger tower at point \( B. \) A fire at point \( C \) is observed from both towers. If \( \angle BAC \) measures 43° and \( \angle ABC \) measures 68°, which ranger tower is closer to the fire? How much closer?
   \[ \text{Tower } B \text{ is } 11.0 \text{ km closer than Tower } A. \]
1. **ALTITUDES** In triangle $ABC$, the altitude to side $AB$ is drawn.

   ![Diagram]

   Give two expressions for the length of the altitude in terms of $a$, $b$, and the sines of the angles $A$ and $B$.

   $$a \sin B = b \sin A = \text{length of altitude}$$

2. **MAPS** Three cities form the vertices of a triangle. The angles of the triangle are 40°, 60°, and 80°. The two most distant cities are 40 miles apart. How close are the two closest cities? Round your answer to the nearest tenth of a mile.

   **26.1 mi**

3. **PHOTOS** Greg took a photograph of the view from his city apartment. The building on the left is the Rocket Tower and the building on the right is the Cloud Scratcher.

   ![Photo]

   Greg’s camera has a 60° viewing angle. Greg knows that he is 2 miles from the Cloud Scratcher and that the Rocket Tower is 3 miles from the Cloud Scratcher. How far is Greg from the Rocket Tower? Round your answer to the nearest hundredth.

   **3.45 mi**

4. **BOATING** A boat heads out to sea from a port that sits along a straight shoreline. The boat heads in a direction that makes a 70° angle with the shoreline. After sailing for 3 miles, the skipper looks back at the shore and sees his house. The house, like the port, also sits on the shore. The lines of sight to the port and to his home make an 80° angle. How far is the skipper’s home from the port? Round your answer to the nearest tenth of a mile.

   ![Diagram]

   **5.9 mi**

5. **ISLANDS** For Exercises 5 and 6, use the following information.

   Oahu is a Hawaiian Island. Off of the coast of Oahu, there is a very tiny island known as Chinaman’s Hat. Keoki and Malia are observing Chinaman’s Hat from locations 5 kilometers apart. Use the information in the figure to answer the following questions.

   5. How far is Keoki from Chinaman’s Hat? Round your answer to the nearest tenth of a kilometer.

   **2.0 km**

   6. How far is Malia from Chinaman’s Hat? Round your answer to the nearest tenth of a kilometer.

   **4.0 km**
8-7 Study Guide and Intervention

The Law of Cosines

The Law of Cosines Another relationship between the sides and angles of any triangle is called the Law of Cosines. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

Law of Cosines

<table>
<thead>
<tr>
<th>Law of Cosines</th>
<th>( \triangle ABC ) be any triangle with ( a, b, ) and ( c ) representing the measures of the sides opposite the angles with measures ( A, B, ) and ( C, ) respectively. Then the following equations are true.</th>
</tr>
</thead>
</table>
| \( a^2 = b^2 + c^2 - 2bc \cos A \) | \( b^2 = a^2 + c^2 - 2ac \cos B \) | \( c^2 = a^2 + b^2 - 2ab \cos C \)

Example 1

In \( \triangle ABC, \) find \( c. \)

\[
\begin{align*}
c^2 &= a^2 + b^2 - 2ab \cos C \\
&= 12^2 + 10^2 - 2(12)(10)\cos 48^\circ \\
c &= \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48^\circ} \\
c &\approx 9.1
\end{align*}
\]

Example 2

In \( \triangle ABC, \) find \( m\angle A. \)

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos A \\
7^2 &= 5^2 + 8^2 - 2(5)(8)\cos A \\
49 &= 25 + 64 - 80 \cos A \\
-40 &= -80 \cos A \\
\frac{1}{2} &= \cos A \\
\cos^{-1}\left(\frac{1}{2}\right) &= A \\
60^\circ &= A
\end{align*}
\]

Exercises

Find each measure using the given measures from \( \triangle ABC. \) Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If \( b = 14, \) \( c = 12, \) and \( m\angle A = 62, \) find \( a. \) \( 13.5 \)

2. If \( a = 11, \) \( b = 10, \) and \( c = 12, \) find \( m\angle B. \) \( 51 \)

3. If \( a = 24, \) \( b = 18, \) and \( c = 16, \) find \( m\angle C. \) \( 42 \)

4. If \( a = 20, \) \( c = 25, \) and \( m\angle B = 82, \) find \( b. \) \( 29.8 \)

5. If \( b = 18, \) \( c = 28, \) and \( m\angle A = 50, \) find \( a. \) \( 24.3 \)

6. If \( a = 15, \) \( b = 19, \) and \( c = 15, \) find \( m\angle C. \) \( 51 \)
8-7 Study Guide and Intervention (continued)

The Law of Cosines

Use the Law of Cosines to Solve Problems You can use the Law of Cosines to solve some problems involving triangles.

Law of Cosines

Let \( \triangle ABC \) be any triangle with \( a, b, \) and \( c \) representing the measures of the sides opposite the angles with measures \( A, B, \) and \( C, \) respectively. Then the following equations are true.

\[
\begin{align*}
\frac{c^2}{a^2} &= b^2 + c^2 - 2bc \cos A \\
\frac{b^2}{c^2} &= a^2 + c^2 - 2ac \cos B \\
\frac{a^2}{b^2} &= a^2 + b^2 - 2ab \cos C
\end{align*}
\]

Example

Ms. Jones wants to purchase a piece of land with the shape shown. Find the perimeter of the property.

Use the Law of Cosines to find the value of \( a. \)

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\begin{align*}
a^2 &= 300^2 + 200^2 - 2(300)(200) \cos 88^\circ \\
a &= \sqrt{130,000 - 120,000 \cos 88^\circ}
\end{align*}
\]

\[\approx 354.7\]

Use the Law of Cosines again to find the value of \( c. \)

\[
c^2 = a^2 + b^2 - 2ab \cos C
\]

\[
\begin{align*}
c^2 &= 354.7^2 + 300^2 - 2(354.7)(300) \cos 80^\circ \\
c &= \sqrt{215,812.09 - 212,820 \cos 80^\circ}
\end{align*}
\]

\[\approx 422.9\]

The perimeter of the land is \( 300 + 200 + 422.9 + 200 \) or about 1223 feet.

Exercises

Draw a figure or diagram to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. A triangular garden has dimensions 54 feet, 48 feet, and 62 feet. Find the angles at each corner of the garden.

\[75^\circ; 48^\circ; 57^\circ\]

2. A parallelogram has a 68° angle and sides 8 and 12. Find the lengths of the diagonals.

\[11.7; 16.7\]

3. An airplane is sighted from two locations, and its position forms an acute triangle with them. The distance to the airplane is 20 miles from one location with an angle of elevation 48.0°, and 40 miles from the other location with an angle of elevation of 21.8°. How far apart are the two locations?

\[50.5 \text{ mi}\]

4. A ranger tower at point \( A \) is directly north of a ranger tower at point \( B. \) A fire at point \( C \) is observed from both towers. The distance from the fire to tower \( A \) is 60 miles, and the distance from the fire to tower \( B \) is 50 miles. If \( m\angle ACB = 62^\circ \), find the distance between the towers.

\[57.3 \text{ mi}\]
8-7 Word Problem Practice

The Law of Cosines

1. **RIGHT TRIANGLES** Triangle $ABC$ is a right triangle with right angle at $B$. Let $c$ be the length of the side opposite $A$, $b$ be the length of the side opposite $B$, and $c$ be the length of the side opposite $C$.

   ![Triangle Diagram]

   Rewrite the Law of Cosines with respect to the right angle $B$ in simplest form.
   \[ b^2 = a^2 + c^2 \] (the Pythagorean Theorem)

2. **LANDSCAPING** Hanna wants to fence a triangular lot as shown. What is the length of the missing side? Round your answer to the nearest foot.

   ![Triangle Diagram]

   \[ 104 \text{ ft} \]

3. **STATUES** Gail was visiting an art gallery. In one room, she stood so that she had a view of two statues, one of a man, and the other of a woman. She was 40 feet from the statue of the woman, and 35 feet from the statue of the man. The angle created by the lines of sight to the two statues was 21°. What is the distance between the two statues? Round your answer to the nearest tenth.

   \[ 14.5 \text{ ft} \]

4. **CARS** Two cars start moving from the same location. They head straight, but in different directions. The angle between where they are heading is 43°. The first car travels 20 miles and the second car travels 37 miles. How far apart are the two cars? Round your answer to the nearest tenth.

   \[ 26.2 \text{ mi} \]

5. **CITIES** For Exercises 5-7, use the following information.

   The cities of Denver, Oklahoma City, and Albuquerque form the vertices of a triangle.

   ![Triangle Diagram]

   Use the information in the figure and round your answers to the nearest tenth of a degree.

   5. What is the measure of the angle at Albuquerque?
   \[ 69.9^\circ \]

   6. What is the measure of the angle at Oklahoma City?
   \[ 38.3^\circ \]

   7. What is the measure of the angle at Denver?
   \[ 71.8^\circ \]